CHAPTER 10

Secondary Structure

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CHAPTER 10

Secondary Structure

Introduction

1 Secondary structure is defined in Chapter 2 specifically as structure which is withstanding secondary loads. Much structure is, of course, withstanding both primary and secondary loads, and the design of that is covered in Chapters 7, 8 and 9. This chapter therefore describes the means for design of structure which is carrying secondary load only, that is mainly short internal decks and tank tops, minor bulkheads and areas of hull and main deck structure towards the ends of the ship where primary loads are negligible.

2 Rather than deal with the problem by area it is more straightforward to discuss methods of design against types of secondary load, leaving it to the designer to decide which process applied to the structure in question, which will usually be obvious from the context. The loads will, therefore, be covered in the order in which they are most frequently encountered, that is lateral pressure, lateral line and point loads and in-plane loads. Under lateral pressure, however, the important sub-set of design to withstand slamming loads will be included.

3 This chapter therefore discusses the design of orthogonally stiffened grillage structures. For wide panels with a large number of unidirectional stiffeners in the short direction, analysis may usually be simplified to one stiffener with effective plate as described for bulkhead design in Chapter 9. The theory is applicable to open-section stiffeners such as are used commonly in ship structures, for which torsional stiffness is usually negligible and shear deflection may be ignored except for short deep stiffeners. Should closed section box or 'top-hat' section stiffeners be used then reference should be made to aerospace literature in the main, for example ESDU Structures Series Volume 6, Megson (1974) or, especially for reinforced plastic marine structures, Smith (1966). The latter reference discusses the 'folded plate' method of design, which can also be used for unidirectionally stiffened structure should the method described in Chapter 9 be considered insufficiently accurate.

Static Lateral Pressure

4 The loads to be applied can be found from the methods described in Chapters 3 and 4. In this chapter it is assumed that a static or quasi-static pressure is known and that an orthogonally stiffened plated grillage is to be designed to resist the pressure. As discussed in Chapter 7, the structure can be viewed in three ways, first as a panel of plate, then as a single stiffener with some associated plating and finally as a complete grillage.

Plate Design

5 The first consideration, that of a plate panel, is a relatively trivial problem and is dealt with in a number of text books, for example Timoshenko, 'Theory of Plates and Shells' or Roark, 'Formulae for Stress and Strain'. The elastic stresses and deflections may be found from data sheet 18.45 (for clamped edges not free to pull in) and are mainly a function of material properties, b/t and aspect ratio, the more efficient structure being when the aspect ratio is 1.0 (unlike structures in edge compression). It must be remembered, however, that every welded structure will be initially distorted and so elastic deflection is relatively meaningless and even stress is of little significance when membrane tension in the distorted plate is invoked. Consequently, when designing the *plating* of secondary structure elastically it will be acceptable to use simple published theories and to allow stresses up to the yield point without any further margin when using the loads from Chapters 3 and 4. Remember, however, to ensure that the weight of the structure itself is included in the load estimate. Boundary conditions will depend on the nature of the load, but will usually be clamped and, unless the grillage is clearly unrestrained, the edges may be assumed unable to pull in. If the load occurs only occasionally in an accident condition, then the plate may be designed plastically as discussed in paragraph 15 of Chapter 9 using data sheets 18.46 to 18.48; the allowable permanent set will probably be of the order of b/50 for secondary structure.

6 Consideration of the plating will give acceptable values of b/t and consequently b and t within a limited realistic range; b should not be normally less than 500 mm to avoid too much complexity and, except for very large heavy grillages, is unlikely to be greater than 2000 mm; t should never be less than 4 mm *after* deduction of rolling and corrosion allowances, from considerations of general ruggedness (except possibly in very small or fast vessels where weight may be an overriding criterion). If, additionally, an aspect ratio of near 1.0 is taken as a starting point, the span of the stiffeners in both directions will be defined within a narrow range.

Stiffener Design

7 If the grillage is likely to have stiffeners in one direction much stiffer (say by a factor of 10 in bending) than in the other direction as illustrated in *Figure 10.1*, the larger stiffeners can be designed to a good first approximation using simple beam theory. To begin, the less stiff beams are neglected and the stiffer beams are sized as if they acted independently under the pressure load applied to the full stiffener spacing, and with an effective breadth of plating acting in bending with the stiffener of half the stiffener spacing (or, if appropriate to the type of stiffener, using effective breadth curves due to Schade (1951 and 1953) included in Chapter 18 as data sheets 18.49 to 18.51). The actual boundary conditions for the edge of the grillage should be used, and a stress factor of 1.2 against yield stress for a working load, and 1.1 for a test load, should be achieved. That is, referring to *Figure 10.2* (paragraph 22) as an example, if the transverse (R) stiffeners are the stiffer ones, these are designed as independent beams under a load per unit length of pressure times spacing b, and with an effective breadth b/2 (up to a maximum of 40t).

8 The less stiff beams (the S set in *Figures 10.1* and *10.2*) are assumed to bend between the stiffer beams, as shown in *Figure 10.1*, over a length equal to the spacing b, and are sized in a similar way to the stiffer beams, but with pressure over a width a and effective breadth a/2 (maximum 40t), and with one important addition. The deformation of the stiff beams means that

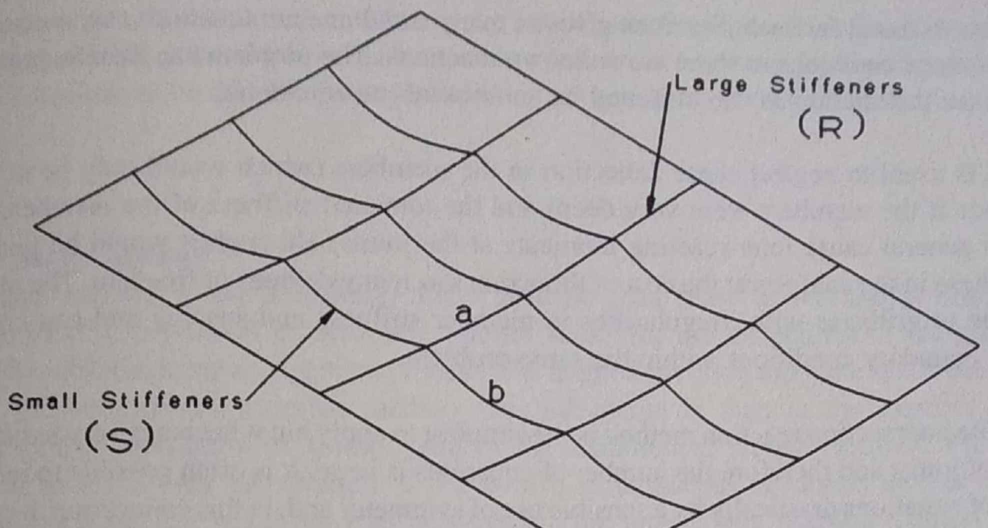

Figure 10.1 Typical Arrangement of Large and Small Stiffeners on a Grillage

each length b of the smaller beams will have a relative deflection between its ends. It is necessary, therefore, to calculate from the deflected shape of the stiff (R) beams what is the worst relative deflection between any two of them along an S beam. This will usually occur in the bay closest to the edge of the grillage and in the line of the centre of the R beams. To estimate this relative deflection it will be adequate to assume a sinusoidal deformation pattern across the grillage parallel to the S beams with a maximum value equal to the central deflection of an R beam as calculated in paragraph 7. That is, if the central deflection is δ_o then the relative deflection δ at a distance a from the edge for a grillage of length A is given by $\delta = \delta_o \sin(\pi a/A)$. This relative deflection is then applied to the ends of a length of the smaller (S) beams in addition to the pressure load and, conservatively, with ends clamped against rotation. The beams should be sized to achieve the same stress factors as for the stiffer beams in paragraph 7.

9 An alternative method, depending on the use of data sheets and some initial estimates of scantlings, is described in the design example at the end of this chapter (paragraphs 36 et seq). However, whatever procedure has been used to size the stiffeners it will be necessary to check the strength of the overall grillage. There are a number of methods which have been developed and a summary of some available analysis procedures and their limitations follows. Further discussion of the methods may be found in Clarkson (1965) and Muckle (1967). In all cases the plating is only included in the analysis as an effective breadth acting in bending with the stiffeners.

Intersection Reaction Method

10 In this method each member of the grillage is considered separately as a beam in simple bending, loaded by the known external loads and by unknown inter-reacting forces at the points where it is crossed by other members. The deflection at each joint is found in terms of these loads, each joint being considered as a point on each of two intersecting members. Evaluating the two deflections so found for each joint then gives as many simultaneous equations as there are joints, that is, as many equations as there are unknown reactions. The problem can then be regarded as solved since the unknowns can be found by solution of the equations.

11 It is usual to neglect shear deflection in the members (which would only be of relative importance if the members were very deep) and the torsional stiffness of the members (which would in general cause inter-reacting moments at the joints) although it would be possible to include these in the analysis at the cost of three times as many degrees of freedom. The method is applicable to grillages with irregularities in member stiffness and spacing and can cope with different boundary conditions within the same problem.

12 The intersection reaction method is the simplest to apply but it becomes very tedious if the number of joints and therefore the number of equations is large. It is often possible to reduce the number of equations drastically by a sensible use of symmetry and, in this connection, it can be an advantage to replace an unsymmetrical load by its symmetrical and antisymmetrical components. Nevertheless, the manual solution of more than about six simultaneous equations is not a task to be lightly undertaken and it is for this reason that the method did not find favour until electronic computing devices became available. In general, however, intersection reaction is an early and simplified version of finite beam element analysis and may be treated and solved as such.

Distributed Reaction Method

13 This method is suitable for use where there are up to about three girders crossed by a large number of equal beams uniformly spaced. The essence of the method is that the discontinuous support provided by the beams to the girders is assumed to be replaced by an equivalent continuous elastic foundation as described, for example, by Roark 'Formulae for Stress and Strain'. The accuracy of the method increases with the number of beams which must all be of equal stiffness and be uniformly spaced. In theory there is no limit to the number of girders but for practical purposes the maximum number is three. The girders need not be uniform, equal nor uniformly spaced. One advantage of this method is that it provides an answer in general form which enables the effect of changes to the beam or girder stiffness to be readily appreciated. Variations in boundary conditions can also be accepted.

Plate Analogy Method

14 In this method the grillage is assumed equivalent to a plate whose stiffness in the direction of the girders is equal to the stiffness of the girders divided by their spacing, and whose stiffness in the direction of the beams is likewise the stiffness of the beams divided by their spacing. The problem thus reduces to the lateral loading of a plate with different stiffnesses in two directions, that is an orthotropic plate. The stiffnesses can be calculated reasonably readily and the theory for analysis of orthotropic plates can be found, for example, in Timoshenko 'Theory of Plates and Shells'.

15 The solution is assumed to be in the form of a Fourier series (described for the in-plane buckling case by Clarke (1977)) whose constants are determined from boundary and loading conditions. For a simply supported grillage this results in finite expressions for deflections and

bending moments, but in the clamped case the result is in the form of an infinite series. This method is rapid and quite simple in the former case but can be laborious for the clamped grillage. It is too laborious to be suitable for asymmetric loads or non-uniform grillages.

Fourier Series Method

16 This method is mathematically more abstruse than those above and is described by Clarkson (1965). It consists of assuming Fourier series for the deflections of the girders and finding the coefficients by an algebraic process. The method involves the same assumptions as are used in the concentrated reaction method. The advantage is that, at the expense of some mathematical complication, it greatly reduces the arithmetical labour required. For a $p \times q$ grillage it involves only the solution of p sets of q simultaneous equations instead of the pq simultaneous equations which have to be solved in the intersection reaction method. For this reason it has been found useful, in the absence of a computer, for tackling the analysis of grillages with too many members for the intersection reaction method to be feasible and too few for the plate analogy method to be very accurate.

Energy Method

17 This method employs the principle of Minimum Potential Energy which is derived from Castigliano's first theorem and which states that the configuration taken up by an elastic body under an applied load is such that the total potential energy of the system is a minimum. A deflected shape is assumed which satisfies the boundary conditions and which is written in the form of a trigonometric series. The constants in the series are then evaluated by equating to zero the partial differential of the total potential energy (that is the strain energy stored in the grillage in deflecting to the assumed shape minus the work done by the loads during that deflection) with respect to each constant in turn. This results in as many simultaneous equations as there are unknown constants.

18 The method has not been widely used. The calculation of deflections is relatively rapid for simply supported grillages with a large number of members in each direction but is laborious for grillages with clamped edges. In both cases the calculation of bending moments and shear forces to sufficient accuracy is tedious.

Relaxation Methods

19 There are several approaches to the solution of grillage problems by relaxation methods, for example see Southwell (1940 and 1941). They are all laborious but do enable every relevant fact to be taken into account (for example torsion, bending, shear and boundary stiffness). For this last reason they have been used to check approximate methods. Unfortunately, each individual case must be worked out separately and so relaxation methods have little to offer now that computer methods are readily available.

Finite Element Methods

20 Probably the most comprehensive method currently available is the use of finite element analysis, discussed in detail for ship structures by Hughes (1983). When using a finite element method, care must be taken to model the bending stiffness of the stiffeners and associated plating correctly. It is still necessary to attach an effective breadth of plating to the stiffeners because shear lag effects are not calculated by the method unless a very refined *plate* mesh is used, with elements allowing for both bending and in-plane effects. It is also important to ensure that the neutral axis of the combined beam-plate elements is correctly positioned in relation to the plane of the nodes and to the outer fibres of the plate and stiffeners so that true bending stresses are calculated. This warning is inserted because many finite element codes assume that the neutral axis of bending is in the plane of the nodes, that is for plated grillages usually in the plane of the plate, unless the data input specifies it to be otherwise, and this specification can be complex.

Graphical Method

21 An alternative to the above methods, appropriate to uniform pressure load cases, was developed by NCRE (now the Defence Research Agency, Dunfermline) in the 1950s and is presented by Clarkson (1965). It uses the intersection reaction method and solutions are presented in a graphical form, included in Chapter 18 as data sheets 18.52 to 18.72. To use this method, the stiffeners in each parallel set must be of equal size and spacing and with the same boundary conditions, although each of these may differ from those of the other parallel set. The overall dimensions of the grillage and the number of stiffeners in each direction are required, using an arrangement similar to that in *Figure 10.2* (similar also to data sheet 18.52). Stiffeners in the more numerous set are designated the R beams (r in number) and those in the less numerous set the S beams (s in number); grillages containing 1, 3, 5, 7, 9 or 15 stiffeners in either direction are specifically considered and results for other numbers can generally be obtained by interpolation.

22 The use of these data sheets is described in detail in Chapter 18 but briefly it involves choosing the relevant data sheets, for example 18.55 and 18.56 give central deflections for all grillages, while 18.57 to 18.60 give bending moments for r × 1 grillages with different boundary conditions, and so on. Then a non-dimensional relative stiffness parameter is calculated based on the geometry and the stiffness of the two types of stiffeners with associated plating. This parameter is input to the curves on the data sheet and a non-dimensional central deflection and a bending moment parameter can be read off, from which the actual values are derived as a function of the applied pressure. Note that, to obtain the highest stress state, the grillage boundary conditions should always be assumed simply supported unless it is certain that the same pressure will be applied to the adjacent grillage at all times.

23 The data curves have been calculated with the uniform pressure load considered as an equivalent concentrated load of magnitude $p \times a \times b$ applied at each stiffener intersection and this is often sufficiently accurate for design purposes. An exception occurs in grillages where the two sets of stiffeners have markedly different stiffnesses. In this case the bending moments in the less stiff members will be significantly affected by local deflection between the relatively rigid supports from the stiffer members at the intersections, and it is necessary to calculate a term additional to that obtained directly from the data sheets. The local bending term is most likely to be significant when the factor ς (= B^3I_r/A^3I_s, where I_r and I_s are second moments of area of the R

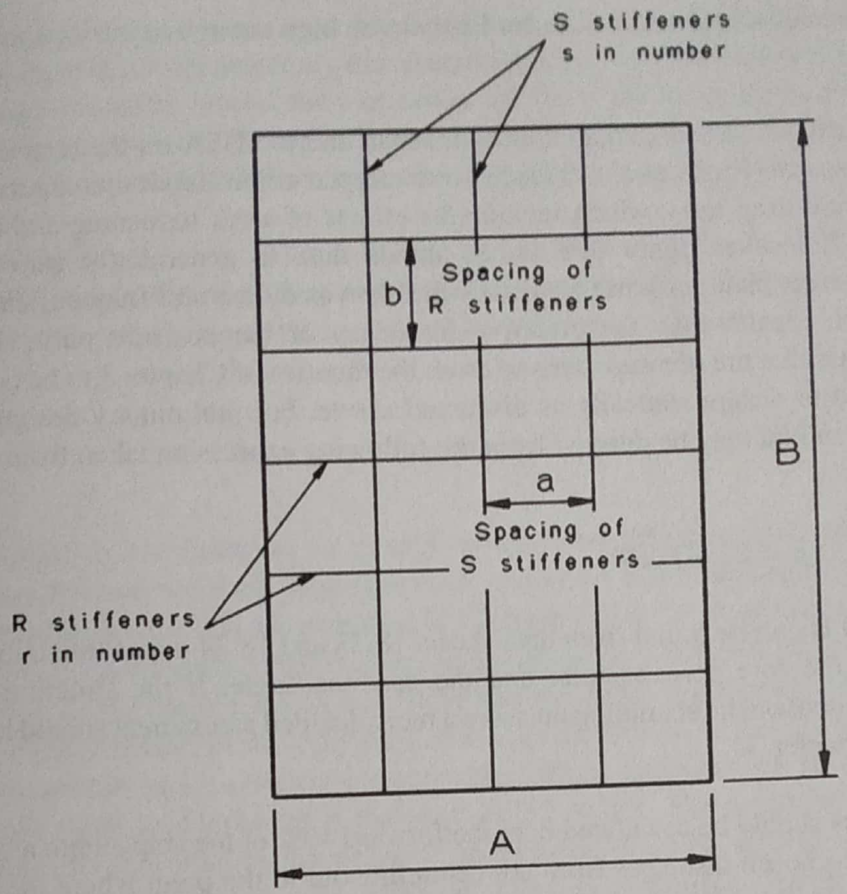

Figure 10.2 Grillage Nomenclature

and S stiffeners with associated plating) is greater than 5 or less than 0.2, but it should always be checked if there is any doubt. The details of how this extra term may be calculated are given in Chapter 18 in the discussion to data sheets 18.53 and 18.54. An example of the use of the graphical method for the design of a grillage is presented at the end of this chapter (paragraph 36 et seq).

Slamming Pressure

24 The prediction of slamming loads has been covered in some detail in Chapter 3 but here it is necessary to discuss the use of those loads in design of the bottom structure. Very high pressures may be generated when the bottom re-enters the water during severe pitching, and as much as 700 kPa has been recorded on destroyers by Bledsoe et al (1960) and damage commensurate with 1000 kPa has been reported on merchant ships by McCallum (1975). Although slamming pressures are most critical on the bottom near the keel it is possible to suffer slamming damage on bow flare and overhanging sponsons, and if the flare angle to the vertical is greater than 30° consideration should be given to local strengthening as described below. Recently dynamic pressures of the order of 50 kPa have been measured in the bow flare area on a Canadian research vessel and reported by Pegg et al (1989). Slamming at the stern in following seas is also possible

but the frequencies and pressures are not likely to be high enough to justify special measures in that area.

25 Studies have been undertaken in both Japan and the USA on the response of plating to impulsive pressures. Nagai et al (1972) have developed empirical design charts for long plates based on vertical drop tests, which include the effects of work hardening and damping due to entrapped air. However, Jones (1973) has shown that, in general, for ship scantlings it is permissible to treat plate response as a static problem as the natural frequencies of the clamped plate are much greater than the effective frequency of the pressure pulse. It is, therefore, reasonable to assume the pressure derived from the theories of Chapter 3 to be constant over the plate panel and to design statically as discussed above. For preliminary design the equivalent static pressure in kPa may be derived from the following expression taken from an unpublished MOD source:

$$p = 22.6\,FGH \tag{10.1}$$

where F, G and H can be found from data sheets 18.73 and 18.74 depending on ship length, the distance abaft the fore perpendicular and the deadrise angle. If the structure appears to be critically dependent on the slamming pressure a more detailed assessment should be carried out as described in Chapter 3.

26 Pressures should be calculated over the forward 40% of the ship length at several buttock lines at suitably chosen distances from the centreline out to the point where the deadrise angle exceeds about 30°. The pressure applied at the keel may be taken to be that calculated at a distance from the centreline equal to 5% of the local waterline beam. Note that the results of such calculations provide an envelope of maximum static equivalent pressures and do not indicate the pressures experienced during any particular slam. Generally the design procedure will be to synthesise the structure using the normal wave loading and then to check the local scantlings against slamming pressures and increase them if necessary.

27 In designing the local bottom structure it should be remembered, as discussed above, that square plate panels are more efficient than long ones in resisting lateral pressure and that, in particular, long plates will develop much larger permanent sets if the design pressure is exceeded. It is, therefore, recommended that the transverse frame spacing should be reduced if necessary over the length of the ship mainly affected by slamming, either overall or by the addition of local intermediate stiffeners, so that the aspect ratio of the plate panels is near to 1.0 and never more than 1.5.

28 There is little published information on the response of the overall bottom grillage to slamming pressures except that it is evident that the effective frequency of the pressure pulse is of the same order as the resonant frequency of the grillage. Further data may be found in the report of Committee II.2 of the ISSC (1988) and earlier ISSC reports. However, there have been no reports of slamming damage to warship bottom *grillages* in recent years except where local detail design has been poor, and so it may be assumed that provided the structure is well designed to resist wave loading then the grillage response to slamming will be elastic. Nevertheless, it will be desirable to check the strength of the stiffeners against about half the equivalent static pressure derived as above so as to ensure that no gross error has been made (the value of half is chosen because slamming pressure will be very unevenly distributed over the whole bottom grillage). Although the bottom grillage is usually curved, the worst case will be for the flatter portions, and a method of analysis for flat grillages may be applied conservatively as described earlier in this chapter. If more detailed checks are required then finite element analysis must be used.

29 The tripping response of stiffeners should be checked using the procedure described in Chapter 7 for lateral loads, but using the *full* pressure derived for the plating design as this is likely to be applied over the whole length of a stiffener between local supports.

Line Loads

30 Line loading is not common on a ship structure but there are two areas where it is significant, along the keel while the ship is in dock and in the waterline region during berthing. Means of estimating these loads are provided in Chapter 4 and for practical purposes they are applied statically. In each case the line load should be applied to a stiffener (if, on the ship's side, the load is applied to the plating, it will deform so that the transverse supporting structure takes the load). It is therefore sufficient to apply the line load in the form of a force per unit length to the stiffener in question taking it as a simple beam with stiffness equivalent to the stiffener plus an effective breadth equal to whichever is the least of b/2 or 40t. The boundary conditions will depend on the extent of the loading, for example on the keel the load is applied over the whole of the ship's length at once so that it will usually be sufficient to assume clamped conditions; for berthing loads, however, there is likely to be some unevenness in the distribution so that pinned ends may be more appropriate. If in doubt always use pinned ends as this condition gives the higher bending moment. The loads given in Chapter 4 are severe ones but as permanent set is undesirable it will be wise to allow a small margin of, say, 1.2 against yielding. Tripping checks should also be carried out as recommended for lateral load in Chapter 7.

31 The line load on the stiffener, or indeed on the plating, is reacted as point loads by the structure at the ends, usually either bulkheads or transverse stiffeners. It will be necessary to check the strength of these members as described in the following paragraphs.

Point Loads

32 Unless they are very small, say less than 10% of the equivalent plate design pressure load ($p \times a \times b$), point loads should never be applied to the middle of the plate panels. Indeed, point loads of any significance should be taken in-plane by bulkheads, decks or the ship's side, and where this is the case the design procedure is given for bulkheads in Chapter 9. The design method for point loads on deck edges is exactly analogous to that for bulkheads.

33 Where point loads are not taken in-plane but are applied to a stiffener some distance from stiff supports, the same procedure may initially be used as suggested for line loads, that is to design the stiffener and associated plating as if it were a simple beam supported at the first stiff structure at its ends. In this case pinned ends should always be assumed. If the load is of significant magnitude, however (say more than 50% of the design pressure load, p × a × b, at an intersection), or if there is any doubt as to the adequacy of the overall structure, then a full grillage analysis will be necessary following one of the procedures listed for pressure load above. Provided that the grillage is not too large, the intersection reaction method is probably be easiest to use in this context especially if torsional stiffness of the members is neglected, in which case the unknowns are the reaction forces and the deflections at each intersection. These unknowns can frequently be reduced in number by judicious use of symmetry and antisymmetry with linear superposition. The effective breadth of plating will be reduced to a fairly small value close to the applied load, but as section modulus is not very sensitive to effective breadth it will be sufficient as before to take the least of b/2 (or a/2) and 40t.

In-Plane Loads

34 In-plane loads are rare on secondary structure except for point loads as discussed above. Should an in-plane load be encountered then the structure should be designed in the same way as for primary structure, as described in Chapters 7 (for distributed loads) and 9 (for point loads). There are, however, two other types of in-plane load which have not yet been discussed, both of which come under the description of edge load reacted by shear. The first of these can occur, for example, in a short flat or deck which is loaded along one edge by pressure on a bulkhead and the load is reacted in shear between the deck and the ship's sides or longitudinal bulkheads. The second case is the more local one of a direct load in a member which is attached along its edge to a plane grillage so that the load is distributed in shear along that edge. Various ingenious analytical methods have been developed in the past, for example Caldwell (1959a and 1959b), but these are laborious to use and involve a number of simplifying assumptions which may be of doubtful validity in particular cases. Consequently, it is best now to use finite element analysis to solve such structural problems as and when they arise.

Stiffener Tripping

35 Although stiffener tripping has been mentioned above in specific cases, it is necessary to check the critical tripping stress of *all* stiffeners under lateral load using the procedure given in Chapter 7 (paragraph 25 and equation 7.10). However, as the failure of a stiffener due to tripping is less critical in secondary structure, a margin of 3 between applied bending stress and critical stress will be adequate. Should the structure be subject to in-plane loads as well as lateral loads then the requirements for resistance to tripping in Chapter 7 (paragraph 18 and equation 7.8) should also be met.

Design Example

36 The example that follows does not include all the topics discussed in this chapter but it does cover the design of a grillage under pressure load which is the principal subject. Specifically, the example is to design an orthogonally stiffened grillage 8 m × 4 m simply supported around all four edges and to support a pressure load of 15 kPa. The design is biassed towards a minimum weight solution; for a discussion of cost minimisation see Chapter 11.

37 Commencing with the plate and referring to Roark 'Formulas for Stress and Strain', case 41 of Table X, for a rectangular plate under uniform pressure (p) with all edges fixed (appropriate for one plate panel amongst many) the maximum stress (σ_b in the centre of the long edge of length b) is given by

$$\sigma_b = \beta p(b/t)^2 \qquad (10.2)$$

where β is a parameter of value 0.31 for a square plate increasing to 0.5 for a long plate of aspect ratio 2 or more, and t is the plate thickness. If as before the material is taken as steel with a yield stress of 280 MPa and if, for the plate, the design is taken up to yield as discussed in paragraph 5, then it is a simple matter to show that for a square plate b/t must be less than 245 and for a long plate less than 193. Note that these values of b/t are much higher than would be appropriate if there was any in-plane loading to contend with.

38 The next stage is to divide the area equally by stiffeners and if initially the more efficient square panels are assumed the following thicknesses can be derived:

9 × 4 stiffeners – 0.8 m × 0.8 m – t > 3.3 mm
7 × 3 stiffeners – 1 m × 1 m – t > 4.1 mm
5 × 2 stiffeners – 1.33 m × 1.33 m – t > 5.5 mm
3 × 1 stiffeners – 2 m × 2 m – t > 8.2 mm.

Of these perhaps only the middle two are suitable for further consideration as the first has plating which is too thin while the last will be very heavy. A few possible rectangular configurations are:

9 × 5 stiffeners – 0.8 m × 0.67 m – t > 3.7 mm
9 × 3 stiffeners – 0.8 m × 1.0 m – t > 4.6 mm
7 × 5 stiffeners – 1.0 m × 0.67 m – t > 5.0 mm
5 × 5 stiffeners – 1.33 m × 0.67 m – t > 6.9 mm
5 × 3 stiffeners – 1.33 m × 1.0 m – t > 6.2 mm.

Of these the 9 × 3 is lightest with an acceptable thickness of plating, but the square aspect ratio 7 × 3 still appears to be the lightest overall and so will be pursued.

39 If the stiffer members are taken in the short direction, which is the most efficient arrangement, then there are seven R beams each of length 4 m. Ignoring, in the first instance, the presence of the S beams and assuming simple beam theory with the beam simply supported, the applied load over a strip 1 m wide is 15 kN/m leading to a maximum bending moment of 30 kN.m; if a margin of 1.2 against yield is required then the section modulus must be more than 129 cm^3. Referring to data sheets 18.32 and 18.33 it will be seen that, for any thickness of plating, a type 3 Tee is too small while a type 4 Tee is significantly too large. There is, therefore, a case for investigating bulb flats or unequal angles; however, a perusal of BS4848 (or data sheet 18.2) shows that although a better match to the required section modulus can be found, the section is always heavier than a type 4 Tee (although possibly cheaper). A better solution might be to change the configuration so that a type 3 Tee is adequate.

40 Returning to the lists above it will be seen that a 9 × 3 arrangement can still just accept a nominal 6 mm plating thickness after deduction of 1.3 mm rolling tolerance, while repeating the calculation above leads to a bending moment of only 24 kN.m and a required section modulus of 103 cm^3 which can now be achieved using type 3 Tees. It should also be noted that 9 × 4m of type 3 Tee is slightly lighter in weight than 7 × 4m of type 4 Tee.

41 To use the graphical method described above and in more detail in Chapter 18 with data sheets 18.52 to 18.72 it is next necessary to calculate the three parameters ζ, μ and ε from equations 10.3 as follows and as mentioned in paragraph 21:

(10.3) $\zeta = B^3 I_r / A^3 I_s$

(10.3a) $\varepsilon = (r + 1)\,\zeta$

(10.3b) $\mu = \varepsilon/(s + 1)$

where in this case r = 9, s = 3, A = 4m, B = 8 m and I_r for a type 3 Tee with 40t (= 190 mm) of plate is 750 cm^4. However, I_s is not yet known! As a *first* estimate *only* it is possible to use the expression

(10.4a) $\zeta = B^3/Aab$ for $B > A$

and

(10.4b) $\zeta = Bab/A^3$ for $A > B$

provided that the larger stiffeners are in the short direction. This gives $\zeta = 160$, $\varepsilon = 1600$ and $\mu = 400$.

42 From data sheet 18.62, the basic bending moment (M_s) at an intersection in an S beam for this value of ε and for a 9 × 3 simply supported grillage is -1.92 MN.m. However, as the S beams are clearly going to be very much less stiff than the R beams ($\zeta \gg 5$), it will be necessary, as discussed in paragraph 23 to adjust the maximum bending moment to allow for local bending between intersections. This is done, as described in Chapter 18, by finding whichever is numerically the greatest of the following expressions:

(10.5a) $M_{max} = M_s + 0.106\ Q_s b^2$

(10.5b) $M_{max} = M_s - 0.072\ Q_s b^2$

(10.5c) $M_{max} = M_s + 0.081\ Q_s b^2$

taking proper account of sign. The first of these gives the bending moment at the first intersection, the second is midway between the edge and the first intersection and the third is an intersection further towards the centre. The parameter Q_s is an equivalent line load on the beam given by 0.625 pb for the particular values of a and b in this grillage. The complete expressions for Q_s (and Q_r when appropriate) are given in Chapter 18.

43 In this case equation 10.5c gives the largest absolute value of bending moment of 1.52 MN.m which, for a margin of 1.2 on yield, leads to a required section modulus for the S beams of 6.6 cm^3. This is very small when compared with what is available either in Tee sections or bulb flats, so a flat bar should be sufficient. Even with a flat bar it can be a waste of time to be too precise as it is always desirable to provide an allowance against poor manufacture for small components; erring therefore a little on the heavy side it can be shown that a 60 mm × 8 mm flat bar with 190 mm of 4.7 mm (6 mm nominal) plate has a section modulus of 7.88 cm^3 (note that neither 50 × 6 mm nor 50 × 8 mm would be stiff enough).

44 I_s is then 41.2 cm^3 which leads to a true value of ζ of 145, with $\varepsilon = 1456$ and $\mu = 364$. The curves in data sheet 18.62 are fairly flat in this range of ε so there is no significant change in the maximum bending moment in the S beams. The actual value of section modulus for the flat bar above gives a maximum bending stress of 194 MPa which is well within the acceptable margin of 1.2 on yield.

45 Returning to the R beams, from data sheet 18.61 for a 9 × 3 grillage with pinned boundaries, M_r is -25.4 kN.m (compared with the estimate of 24 in paragraph 39 - the sign was not applied as no sign convention was used for the initial estimate. In fact, a negative bending moment implies tension in the fibres most distant from the applied pressure). A type 3 Tee with 190 mm of 6 mm (nominal) plating has a section modulus of 119 cm^3 giving a maximum stress of 214 MPa; note that for the heavier stiffeners there is no correction for local bending and so this is the actual maximum, and again it is within the acceptable margin on yield.

46 A check on critical tripping stresses for both stiffeners shows that they are well within the allowable range. The final design of the grillage is then 8 m × 4 m of 6 mm nominal thickness plate with 9 in number type 3 Tee stiffeners at 800 mm spacing in the short direction and 3 in number 60 mm × 8 mm flat bar intercostals at 1000 mm spacing in each bay in the long direction. The average weight of this grillage is 6.25 kg/m^2 giving a load of 0.65 kPa which must, of course, be deducted from the assumed applied pressure of 15 kPa when assessing its ability to include all the actual loads on the structure.